MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

EEM3066 – RANDOM PROCESSES & QUEUEING THEORY

(TE)

15 OCTOBER 2016

9:00 A.M – 11:00 A.M.

(2 Hours)

INSTRUCTIONS TO STUDENT:

1. This exam paper consists of **9 pages** with **4 questions**.
2. Attempt **ALL questions**. All questions have equal weightage and the breakdown of marks within each question is given.
3. Please write all your answers in the answer booklet provided. Show all relevant steps to obtain maximum marks
4. Some common trigonometric identities, Fourier transform pairs, Fourier transform properties and the Erlang-B table are supplied in the **Appendix**.

Question 1

(a) Consider two random variables $\boldsymbol{X}$ and $\boldsymbol{Y}$ with joint pdf

$$f(x,y) = \begin{cases} 6x^2y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & elsewhere \end{cases}$$

i. Show that $f(x,y)$ is a valid pdf.

[3 marks]

ii. Find the marginal pdf for X and the marginal pdf for Y.

[4 marks]

iii. Find the mean and variance of X.

[4 marks]

iv. What is the probability that $X > Y$?

[5 marks]

(b) The joint pdf of two random variables X_1 and X_2 is given by

$$f(x_1,x_2) = \begin{cases} 2e^{-x_1}e^{-x_2} & 0 < x_1 < x_2 < \infty \\ 0 & elsewhere \end{cases}$$

Consider the transformation $Y_1 = 2X_1$ and $Y_2 = X_2 - X_1$.

i. Find the joint pdf $f(y_1,y_2)$.

[5 marks]

ii. Show that Y_1 and Y_2 are independent.

[4 marks]

Continued...

Question 2

Consider a linear time-invariant system with impulse response $h(t)$. Suppose the input of the system is a wide-sense stationary (WSS) process $X(t)$ as shown in Figure Q2.1.

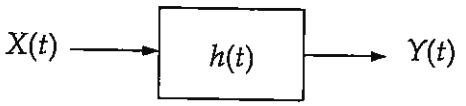

Figure Q2.1

i. Show that $Y(t)$ is WSS.

[9 marks]

Assume that the input $X(t)$ is a zero-mean Gaussian random process with

$$S_{XX}(f) = \eta/2$$

and the impulse response of the system is

$$h(t) = \begin{cases} \exp(-t) & t \geq 0 \\ 0 & elsewhere \end{cases}$$

ii. Find $S_{YY}(f)$.

[5 marks]

iii. Compute the average power of $Y(t)$.

[6 marks]

Assume that the signal $Y(t)$ is now passed through a whitening filter $g(t)$ such that the output $W(t)$ is a white Gaussian process as illustrated in Figure Q2.2.

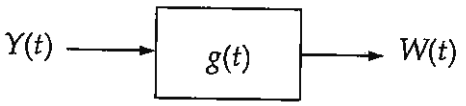

Figure Q2.2

iv. Determine the required transfer function of the whitening filter $G(f)$.

[5 marks]

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Question 3

(a) A typical communication system model that employs a matched filter is shown in Figure Q3

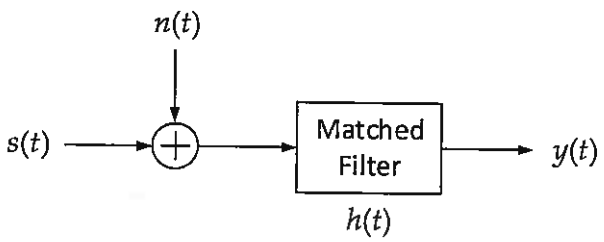

Figure Q3

where $s(t)$ is the desired signal and $n(t)$ is white Gaussian noise with power spectral density $N_0/2$ W/Hz. Assume that $s(t)$ is given by

$$s(t) = \begin{cases} e^{2t} & -1 < t < 0 \\ 0 & elsewhere \end{cases}$$

i. Find the impulse response of the matched filter $h(t)$ with time delay t_0.
[4 marks]

ii. What is the suitable value for the time delay t_0 so that the filter is causal?
[2 marks]

iii. Let $t_0 = 1$. Find the average power of the signal (assume $n(t) = 0$) at the output of the matched filter at $t = 1$.
[8 marks]

iv. Find the maximum SNR at the output of the matched filter.
[3 marks]

(b) Consider a random process $X(t)$ corrupted by an uncorrelated noise $N(t)$. The autocorrelation functions of these two processes are given as follows:

$$R_{XX}(\tau) = \frac{1}{2}\exp(-|\tau|) \text{ and } R_{NN}(\tau) = \frac{N_0}{2}\delta(\tau)$$

i. Find the transfer function of the Wiener filter.
[6 marks]

ii. What is the error criterion for Wiener filter?
[2 marks]

Continued...

Question 4

(a) Customers arrive at a fast food restaurant according to a Poisson process of rate λ customers per hour. The restaurant has c employees to attend to the customers. Suppose that it takes an employee an exponentially distributed amount of time with mean $1/\mu$ hours to service one customer.

i. State (using Kendall's notation) the type of queueing system described above. [3 marks]

ii. Draw the transition rate diagram. [5 marks]

(b) Consider a M/M/1 queueing system with customer arrival rate of 2 per minute.

i. How fast should the service rate be if it is required that less than 5 customers line up 90% of the time? [Hint: $p_j = (1-\rho)\rho^j$, $\sum_{i=0}^{n} c^i = \frac{1-c^{n+1}}{1-c}, for\ c \neq 1$]

[8 marks]

ii. If the average service time is 15 seconds per customer, what is the probability that an arriving customer will find the counter occupied?

[6 marks]

iii. Assume the same average service time as in Part (ii), what is the average time spent by a customer in the system? [Hint: $E[N] = \frac{\rho}{1-\rho}$]

[3 marks]

Continued...

APPENDIX A: Common Trigonometric Identities

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin(x)$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$2\cos(x) = e^{jx} + e^{-jx}$$

$$2j\sin(x) = e^{jx} - e^{-jx}$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$

$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$$

$$2\cos^2(x) = 1 + \cos(2x)$$

$$2\sin^2(x) = 1 - \cos(2x)$$

Continued...

APPENDIX B: Common Fourier Transform Pairs

Time	Frequency
$\delta(t)$	1
1	$\delta(f)$
$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$
$sign(t)$	$\frac{1}{j\pi f}$
$\exp(j2\pi f_0 t)$	$\delta(f-f_0)$
$\exp(-\alpha t)u(t)$	$\frac{1}{\alpha+j2\pi f}$
$\exp(-\alpha\lvert t\rvert)$	$\frac{2\alpha}{\alpha^2+(2\pi f)^2}$
$t\exp(-\alpha t)u(t)$	$\left(\frac{1}{\alpha+j2\pi f}\right)^2$
$\frac{t^{n-1}}{(n-1)!}\exp(-\alpha t)u(t)$	$\left(\frac{1}{\alpha+j2\pi f}\right)^n$
$\exp(-\alpha^2 t^2)$	$\sqrt{\pi/\alpha^2}\exp\left(-\frac{\pi^2 f^2}{\alpha^2}\right)$
$\cos(2\pi f_0 t)$	$\frac{1}{2}[\delta(f-f_0)+\delta(f+f_0)]$
$\sin(2\pi f_0 t)$	$\frac{1}{2j}[\delta(f-f_0)-\delta(f+f_0)]$
$A\,rect\left(\frac{t}{\tau}\right)$	$A\tau\,sinc(f\tau)$
$sinc(2Wt)$	$\frac{1}{2W}rect\left(\frac{f}{2W}\right)$

Continued...

APPENDIX C: Fourier Transform Properties

Operation	Time	Frequency
Addition	$x_1(t) + x_2(t)$	$X_1(f) + X_2(f)$
Scalar multiplication	$kx(t)$	$kX(f)$
Duality/Symmetry	$X(t)$	$x(-f)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Time shift	$x(t - t_0)$	$X(f)\exp(-j2\pi f t_0)$
Frequency shift	$x(t)\exp(j2\pi f_0 t)$	$X(f - f_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(f)X_2(f)$
Frequency convolution	$x_1(t)x_2(t)$	$X_1(f) * X_2(f)$
Time differentiation	$\frac{d^n}{dt^n}x(t)$	$(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
Conjugation	$x^*(t)$ $x^*(-t)$	$X^*(-f)$ $X^*(f)$

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APPENDIX D: Erlang-B Table

Maximum Offered Load Versus B and N
B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84

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